Practice Set 8 Probability Part II Multiplication Rules

I. Below is the data Darin Jones collected concerning sales to customers of different ages. (see page 42)
Convert Table 1 to decimals and place the information into Table 2.

Analysis of Sales By Age of Customer (Table 1))				Decimal Analysis (Table 2)		
Customer Age / Sale	Less than or equal to 20	Over 20	Totals	Less than or equal to 20	Over 20	Totals
No	16	8	24	0.267	0.133	0.40
Yes	24	12	36	0.400	0.200	0.60
Totals	40	20	60	0.667	0.333	1.00

II. Use a formula to calculate the probability of these events and check your answers using Table 2.

A. The probability of a customer being over 20 years old.

$P(>20) = \frac{\geq 20}{n} = \frac{20}{60} = .333 \rightarrow 33.3\%$ Note how 33.3% can be read directly from Table 2.

B. The probability of a customer being over 20 years old and not making a sale.

$P(>20 \text{ and } \tilde{S}) = P(>20)\, P(\tilde{S}) = \frac{20}{60} \times \frac{24}{60} = \frac{480}{3{,}600} = .133 = 13.3\%$ See Table 2

C. The probability of a customer being less than or equal to 20 years old and over 20 years old. These events are mutually exclusive, their intersection is empty. $P(\leq 20 \text{ and } > 20) = 0$

D. Was the special rule of multiplication applicable to question B? Why or why not? Could the special rule of multiplication be used by Linda with the page 46 advertising data? Why or why not?

1. The special rule is appropriate because the events are independent. Age does not affect buying habits as demonstrated by the fact that 60% of both age groups make a purchase.
2. The special rule for multiplication is not appropriate for the page 46 problem because sales and advertising are dependent.

III. Use Bayes' theorem to calculate the probability of making a sale given a customer is less than or equal to 20 years of age.

$$P(S \mid \leq 20) = \frac{P(S \text{ and } \leq 20)}{P(\leq 20)} = \frac{P(S) \times P(\leq 20 \mid S)}{P(S) \times P(\leq 20 \mid S) + P(\tilde{S}) \times (P \leq 20 \mid \tilde{S})} = \frac{\frac{36}{60} \times \frac{24}{36}}{\frac{36}{60} \times \frac{24}{36} + \frac{24}{60} \times \frac{16}{24}} = \frac{.40}{.40 + .267} = .60 = 60\%$$

IV. Recalculate your answer to question III using Table 2 on page 48. $.400 \div .667 = .6$ or 60%

V. Use Linda's page 46 advertising data to calculate the possibility of having monthly advertising over $5,000 and monthly sales over $50,000.

$$P(A > \$5{,}000 \text{ and } S > \$50{,}000) = P(S > \$50{,}000)\, P(A > \$5{,}000 \mid S > \$50{,}000)$$
$$\frac{5}{10} \times \frac{4}{5} = \frac{20}{50} = 40\%$$

VI. Answer these questions about 5 posters Darin has to advertise a new CD recorder/player.

A. How many ways can he arrange these posters in a horizontal line across a wall?

$N! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ arrangements

B. How many ways can he arrange only 3 posters? Arrange implies that order counts. AB is not the same as BA and that both should be counted.

$${}_N P_R = \frac{N!}{(N-R)!} \qquad {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 5 \times 4 \times 3 = 60$$

C. How many ways can he just hang them? (order doesn't count)

$${}_N C_R = \frac{N!}{(N-R)!(R!)} \qquad {}_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} = \frac{5 \times 4}{2 \times 1} = 10$$